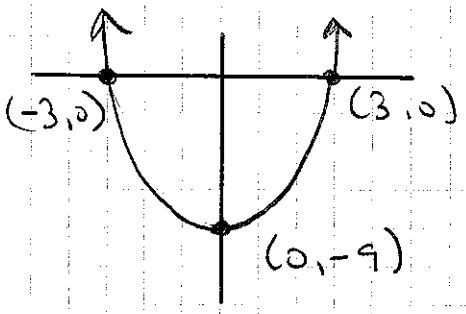
Supplemental Worksheet 12

① • draw $y = x^2 - 9$



VA @ $x = \pm 3$

HA @ $y = 0$

invariant

$y = 1$

$1 = x^2 - 9$

$x^2 = 10$

$x = \pm\sqrt{10}$

$(-\sqrt{10}, 1)$ $(\sqrt{10}, 1)$

$y = -1$

$-1 = x^2 - 9$

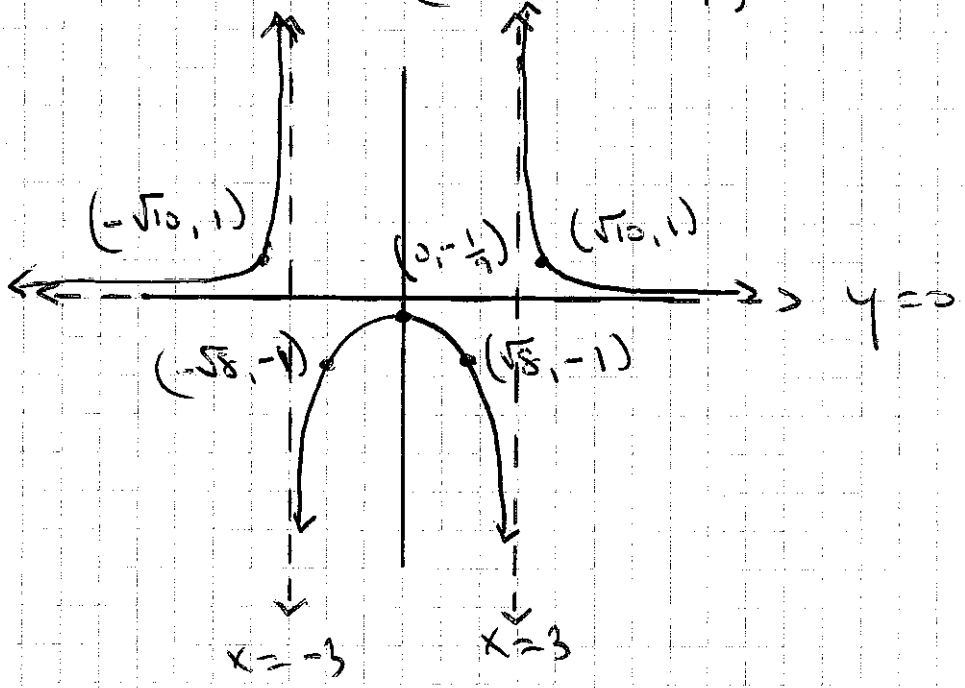
$x^2 = 8$

$x = \pm\sqrt{8}$

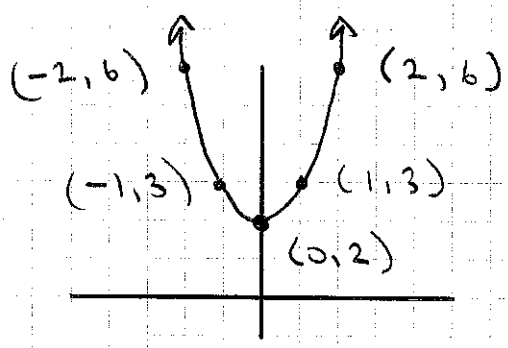
$(-\sqrt{8}, -1)$ $(\sqrt{8}, -1)$

local min @ $(0, -9)$

so local max @ $(0, -\frac{1}{9})$



② draw $y = x^2 + 2$



No VAs (no zeros on ↗)

HA @ $y = 0$

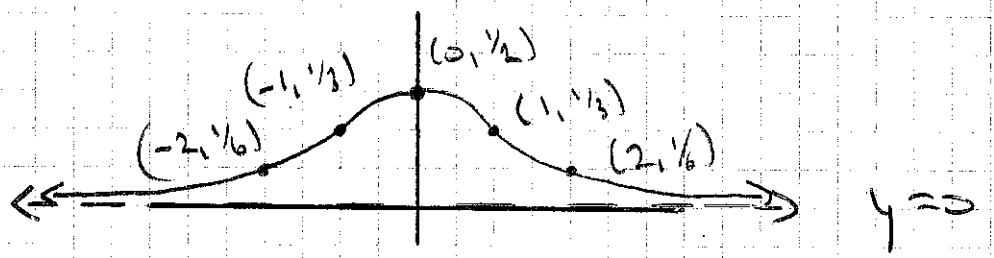
Invariant – none

local min @ $(0, 2)$

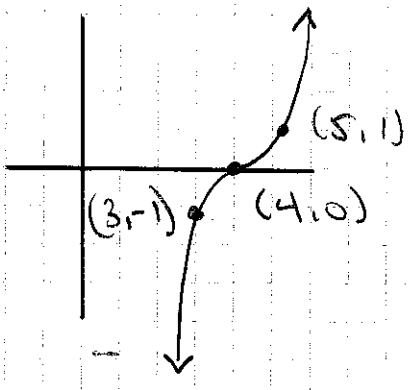
∴ local max @ $(0, \frac{1}{2})$

Some other points:

$(-2, 6)$		$(-2, \frac{1}{6})$
$(-1, 3)$	→	$(-1, \frac{1}{3})$
$(1, 3)$		$(1, \frac{1}{3})$
$(2, 6)$		$(2, \frac{1}{6})$



③ draw $y = (x-4)^3$



VA at $x = 4$

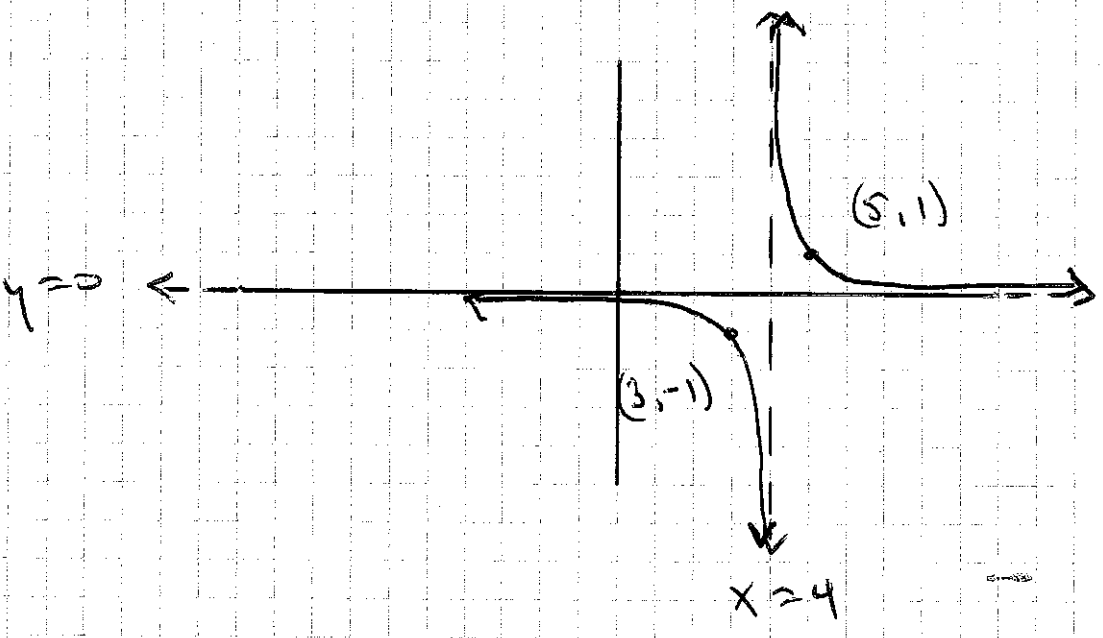
HA at $y = 0$

Invariant

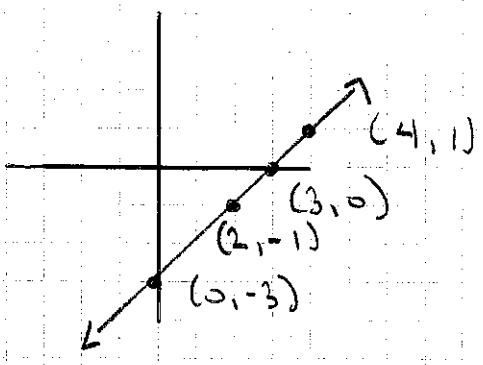
$y = 1$ ↓ $(5,1)$

$y = -1$ ↓ $(3,-1)$

no local min or max



④ draw $y = x - 3$



VA @ $x = 3$

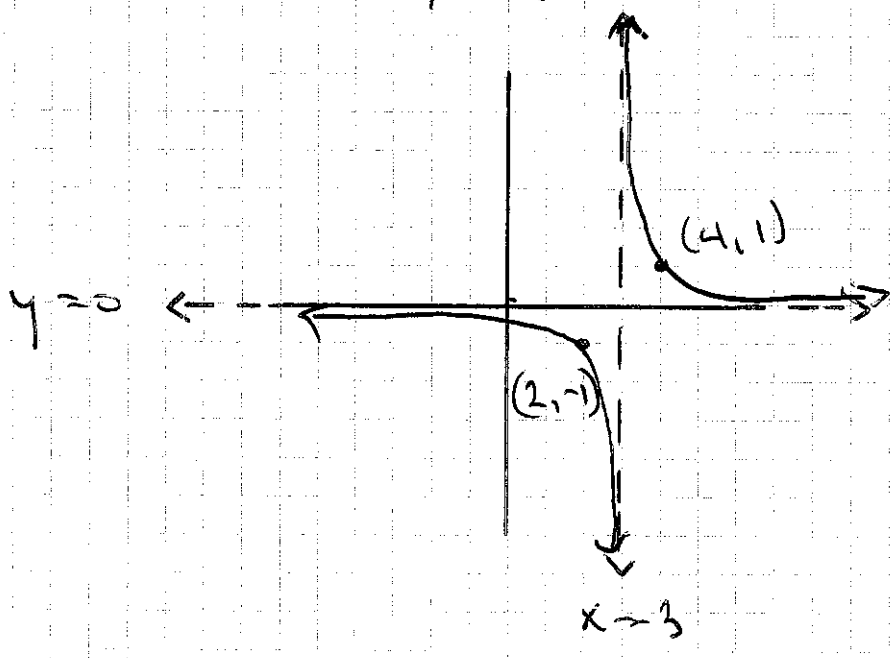
HA @ $y = 0$

Invariant

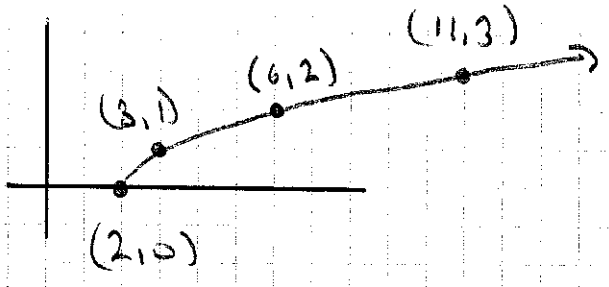
$y = 1$ ↓ $(4,1)$

$y = -1$ ↓ $(2,-1)$

no local min / max



(5) draw $y = \sqrt{x-2}$



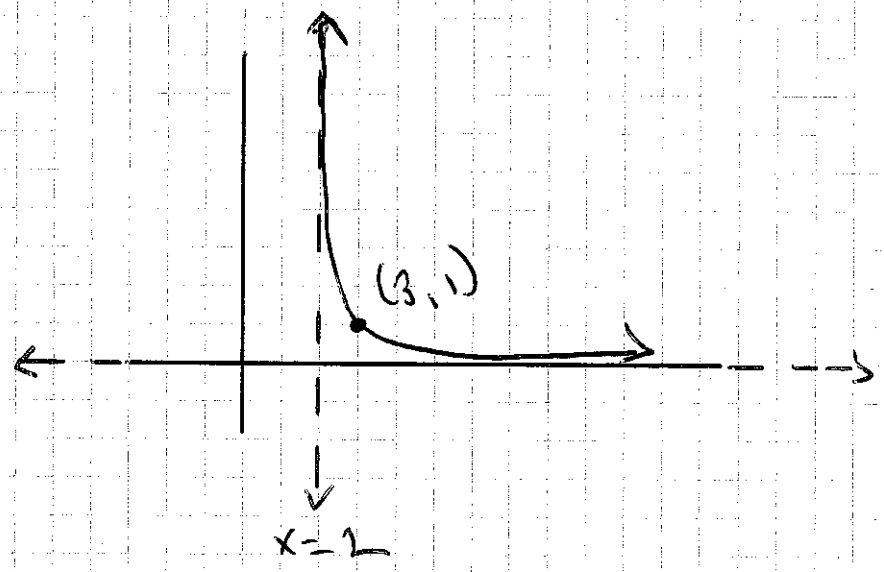
VA @ $x = 2$

HA @ $y = 0$

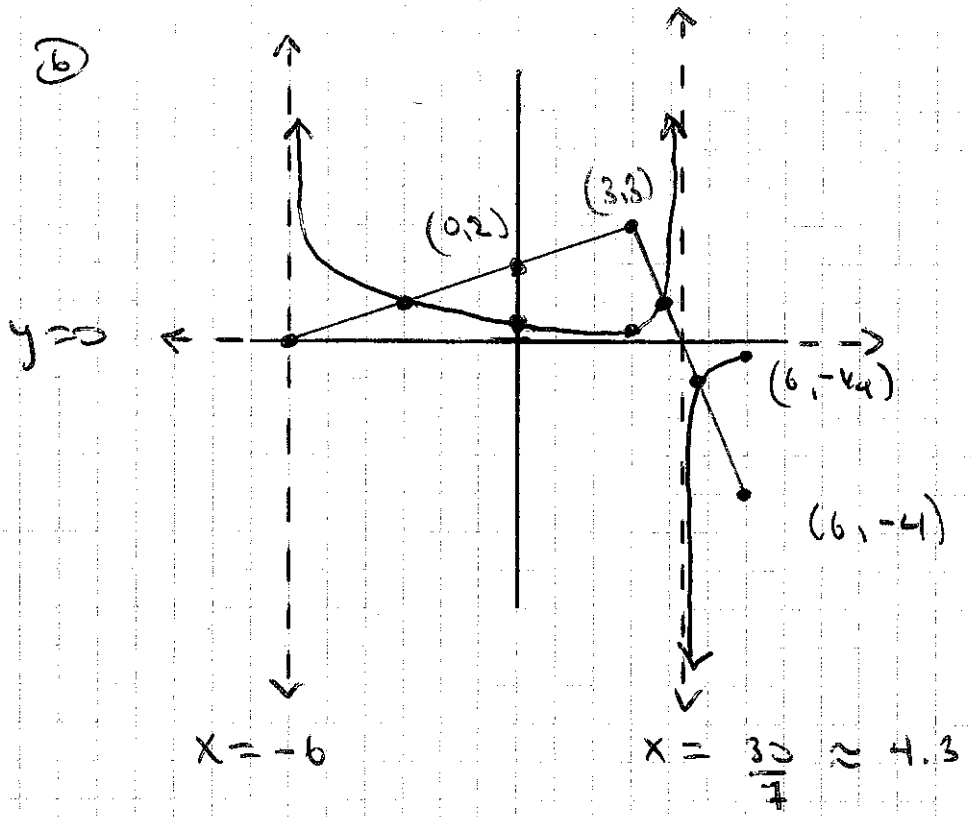
invariant

$y = 1 \longrightarrow (3,1)$

no local min / max



⑥



VA @ zeros

HA @ $y = 0$

Invariant

$y = 1$ ↙ ↘ $y = -1$ ↙

$(-3, 1)$ $(\frac{27}{7}, 1)$ $(\frac{33}{7}, -1)$

↑ 3.9 ↑ 4.7

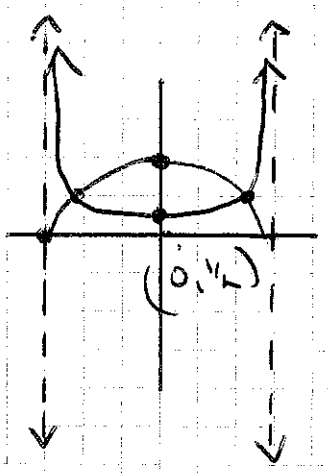
no local min / max

Other points $(0, 2) \to (0, \frac{1}{2})$

$(3, 3) \to (3, \frac{1}{3})$

$(6, -4) \to (6, -\frac{1}{4})$

(7)



VA @ zeros

no HA

Invariant

$y = 1 \longrightarrow$ at approx ± 2

local max @ $(0, 2)$

$\therefore$ local min @ $(0, \frac{1}{2})$